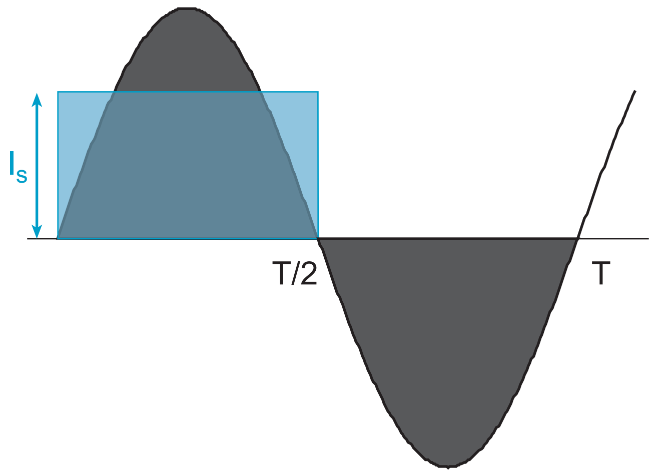
Mean value

Definition: $I_S = \frac{1}{T}\int_{t_0}^{T+t_0} i(t)dt$

Meaning: The value of a DC current *I*, that transfers the same charge. The value is corresponding to the height of a rectangle of equal area as waveform of current *i*(*t*) during one period *T*.

In contrast to e.g. Fourier series the meaning of the mean value is not a DC component of a waveform, but it is the total possible effect of the electric current regardless of the sign of the instantaneous value. Therefore is necessary compute the mean value by means of an **absolute value** (**mean rectified value**), or, in the case of symmetrical waveforms where the areas of both half-periods (quarter-periods, …) are the same (except the sign) we can calculate the **mean value in a half-period** (a quarter-period, …) only. The height of a rectangle will be the same!



Root mean square

Definition: $I_{RMS} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt}$,

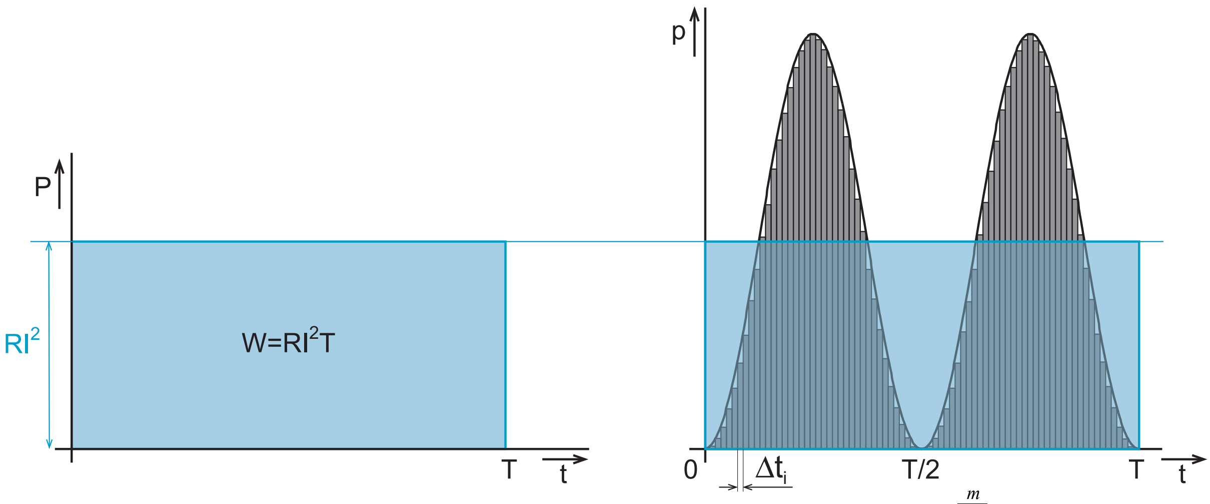
In Czech books could be just symbol *I*. Measuring devices, which measure directly rms value, could be denoted by RMS symbol, or true RMS (but "RMS calibrated" signifies the mean value is measured and multiplied by 1.11 to show RMS value – approach valid only for sinusoidal time function). The root mean square term is very descriptive, see the definition equation…

Meaning: The value of the DC current *I*, which generates an equivalent amount of heat like alternating current

Derivation:

1. Amount of heat, generated by a DC current within a period T: $W = RI^2T$
2. Instantaneous power of a alternating current: $p = Ri^2(t)$
3. Total amount of heat, generated by an alternating current within a period T could be evaluated by the "sum" of instantaneous powers – that is by the integration (see figure) $W = \int_0^T Ri^2(t)dt$
4. Finally, comparing (1) and (3) we obtain

$$RI^2T = \int_0^T Ri^2(t)dt \qquad \Rightarrow \qquad I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$



$$W = \lim_{m\to\infty}\sum_{i=1}^{m} p_i \Delta t_i$$

Factor definitions

Form factor	$k_t = \frac{I}{I_s}$	Necessary, if we have instrument measuring the mean value and we need the rms value… It is usual, because of the most important question is dissipated heat. Necessary for calculation of induced voltages.
Crest factor (peak-to-rms ratio)	$k_c = \frac{I_m}{I}$	Every instrument measuring the rms value (or power) has its specific allowable crest factor, so, if the crest factor of some waveform is too big, the result of measuring is wrong – it affect the accuracy of the measuring. One reason is limited frequency range of the meters (as you will see next term, the waveform must contain sinusoidal variables of high frequencies), the other could be limited dynamic range. The crest factor is also used in advanced signal analysis, e.g. for impacting detection (roller bearing wears, gear tooth wears, or other rotating devices).
Factor	$k_p = \frac{I_s}{I_m}$	

Selected waveforms:

Sine:	$i(t)=I_m\sin(\omega t)$
Square wave:	$i(t)=I_m \qquad t\in\left\langle 0,\frac{T}{2}\right)$ $=-I_m \qquad t\in\left\langle \frac{T}{2},T\right)$
Triangular wave:	$i(t)=\frac{4I_m}{T}t \qquad t\in\left\langle 0,\frac{T}{4}\right)$ $=2I_m-\frac{4I_m}{T}t \qquad t\in\left\langle 0,\frac{T}{4}\right)$ $=-4I_m+\frac{4I_m}{T}t \qquad t\in\left\langle 0,\frac{T}{4}\right)$

Computed values

	sine	square wave	triangular wave
I_s	$\frac{2I_m}{\pi}$	I_m	$\frac{I_m}{2}$
I	$\frac{I_m}{\sqrt{2}}$	I_m	$\frac{I_m}{\sqrt{3}}$
$k_t=\frac{I}{I_s}$	$\frac{\pi}{2\sqrt{2}}\doteq 1.11$	1	$\frac{2}{\sqrt{3}}=1.15$
$k_c=\frac{I_m}{I}$	$\sqrt{2}$	1	$\sqrt{3}$
$k_p=\frac{I_s}{I_m}$	$\frac{2}{\pi}$	1	$\frac{1}{2}$

Computations

Sine:

$$I_s=\frac{1}{\frac{T}{2}}\int_0^{\frac{T}{2}}I_m\sin(\omega t)dt=\frac{2I_m}{T}\left[\frac{-\cos(\omega t)}{\omega}\right]_0^{\frac{T}{2}}=\frac{2I_m}{T}\left[\frac{-\cos\left(\frac{2\pi}{T}t\right)}{\frac{2\pi}{T}}\right]_0^{\frac{T}{2}}=\frac{I_m}{\pi}(-\cos(\pi)+\cos(0))=\frac{2I_m}{\pi}$$

$$I=\sqrt{\frac{1}{T}\int_0^T(I_m\sin(\omega t))^2dt}=\left|\sin^2(\omega t)=\frac{1-\cos(2\omega t)}{2}\right|=\sqrt{\frac{I_m^2}{2T}\int_0^T(1-\cos(2\omega t))dt}=$$

$$=I_m\sqrt{\frac{1}{2T}\left\{[t]_0^T-\frac{1}{2\omega}[\sin(2\omega t)]_0^T\right\}}=I_m\sqrt{\frac{1}{2T}\left[T-\frac{1}{2\cdot\frac{2\pi}{T}}(\sin(4\pi)-\sin(0))\right]}=\frac{I_m}{\sqrt{2}}$$

Triangular waveform:

$$I_S = \frac{1}{\frac{T}{4}} \int_0^{\frac{T}{4}} \frac{4I_m}{T} t dt = \frac{16I_m}{T} \left[\frac{t^2}{2} \right]_0^{\frac{T}{4}} = \frac{8I_m}{T} \left(\frac{T}{4} \right)^2 = \frac{I_m}{2}$$

$$I = \sqrt{\frac{1}{\frac{T}{4}} \int_0^{\frac{T}{4}} \left(\frac{4I_m}{T} t \right)^2 dt} = \sqrt{\frac{64 I_m^2}{T^3} \left[\frac{t^3}{3} \right]_0^{\frac{T}{4}}} = \frac{I_m}{\sqrt{3}}$$

Note: equations, computations and table of results are introduced for current. Analogical equations are valid also for voltage. Derivation of root mean square of the voltage arises from the equation $W = \frac{U^2}{R} T$.