Measurement on simple RC network

Measurement objectives

1. Connect all instruments and units according to the figure 1:
   • the generator G of the sinusoidal voltage \( u_1(t) \) connect to the input terminals of the RC circuit
   • AC voltmeter (digital multimeter switched to the AC voltage measurement “ACV”) – DMM – connect to the input of the RC circuit
   • to the input and output terminals of the RC circuit connect terminals of the phase meter A and B
   • channel CH1 of the two-channel oscilloscope OSC connect to the input of the RC circuit (voltage \( u_1(t) \)) and channel CH2 to the output (voltage \( u_2(t) \)).

   ![Figure 1](image)

2. Adjust on generator G frequency \( f = 1 \, \text{kHz} \) and RMS value \( U_1 = 10 \, \text{V} \) (measured by the electronic voltmeter DMM) of the sinusoidal voltage \( u_1(t) \).
• When you switch the switcher P to the position 2, the voltmeter will measure RMS value of the output voltage $U_2$.

• Using the phase meter PM, or oscilloscope OSC measure phase shift between input voltage $u_1(t)$ and output voltage $u_2(t)$.

• Compute the amplitude and phase of the transfer function of the RC circuit for given frequency and find its complex representation.

3. Find the frequencies $f_1$, $f_2$, and $f_3$, for which the phase shift between input voltage $u_1(t)$ and output voltage $u_2(t)$ is $\psi_1 = \pi/6$ ($-\pi/6$), $\psi_2 = \pi/4$ ($-\pi/4$) and $\psi_3 = \pi/3$ ($-\pi/3$). Using the procedure described in the step 2 measure and compute values of the amplitude and phase shift and complex representation of the transfer function.

**Theoretical analysis**

If linear electrical circuit is supplied from sinusoidal source with frequency $\omega$ and its energetical conditions are steady, than the circuit is in *sinusoidal steady state*. All circuit variables are sinusoidal waveforms having same frequency $\omega$.

Analysis of circuits in sinusoidal steady state can be significantly simplified by simple transform, which assign to each sinusoidal waveform $x(t)$ with frequency $\omega$ complex number $X_m$. This complex number is called *phasor*:

\[ x(t) = X_m \sin(\omega t + \varphi) \rightarrow X_m = X_m e^{j\varphi}. \tag{1} \]

*Phasor* $X_m$ with its modulus and phase is unique representation of amplitude $X_m$ and initial phase $\varphi$ of sinusoidal waveform $x(t)$. Using inverse transform is possible assign sinusoidal waveform $x(t)$ back to the phasor $X_m$:

\[ x(t) = \text{Im}[X_m e^{j\varphi}] \leftarrow X_m. \tag{2} \]

When the input of linear circuit is supplied from the sinusoidal voltage source $u_1(t)$, on the output (when the circuit is in the steady state):

\[ u_1(t) = U_{1m} \sin(\omega t + \varphi_1) \tag{3} \]

\[ u_2(t) = U_{2m} \sin(\omega t + \varphi_2). \tag{4} \]

Using transformation (1) we can transform input and output voltage on phasors:

\[ U_{1m} = U_{1m} e^{j\varphi_1}, \tag{5} \]

\[ U_{2m} = U_{2m} e^{j\varphi_2}. \tag{6} \]

The voltage transfer function $P$ of the linear circuit is defined as ratio of phasors $U_{2m} / U_{1m}$:

\[ P = \frac{U_{2m}}{U_{1m}}. \tag{7} \]

*The modulus (absolute value) $P$ of the voltage transfer function defines the ratio of amplitudes $U_{2m} / U_{1m}$, or the ratio of RMS values $U_2 / U_1$ of the output and input voltage, while the phase $\psi$ of the voltage transfer function defines the phase shift between output and input voltages $\varphi_2 - \varphi_1$:

\[ P = \frac{U_{2m} e^{j\varphi_2}}{U_{1m} e^{j\varphi_1}} = \frac{U_{2m}}{U_{1m}} e^{j(\varphi_2 - \varphi_1)} = \frac{U_2}{U_1} e^{j(\varphi_2 - \varphi_1)} = P e^{j\psi}. \tag{8} \]
The procedure of the measurement of the voltage transfer function $P$ for distinct frequency $\omega = 2\pi f$ is following:

- Connect sinusoidal voltage source with frequency $\omega = 2\pi f$ to the input of the circuit.
- Measure RMS values of input and output voltages $U_1$ and $U_2$ using voltmeter (used voltmeter measures RMS value of voltage)
- Measure phase shift between input and output voltages using phasemeter (or oscilloscope)
- Evaluate measured values by the equation (8) – table 1.

<table>
<thead>
<tr>
<th>$f$ [Hz]</th>
<th>$U_1$ [V]</th>
<th>$U_2$ [V]</th>
<th>$P$ [-]</th>
<th>$\psi$ [rad]</th>
<th>$P$ [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f_2$</td>
<td></td>
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</tr>
<tr>
<td>$f_3$</td>
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</tbody>
</table>

Table 1

Measurement of the phase shift of two sinusoidal voltages with the aid of the oscilloscope

On the screen of the two-channel oscilloscope we will display two sinusoidal voltage waveforms $u_1(t)$ and $u_2(t)$ with same frequency $\omega$, described by the equations (3) and (4). (Fig. 2)

![Fig. 2](image)

With the aid of vertical cursors (dashed lines on Fig. 2) we marked positions of two points with the same phase shift $\Phi$ on both waveforms (e.g. voltage zero with positive derivation). For the first point (voltage $u_1(t)$) we will read the time position (on the screen) $t_1$, for the second point (voltage $u_2(t)$) time position $t_2$. When the frequency is $\omega$, then we can find the phase shift
between the voltages \( u_1(t) \) and \( u_2(t) \) \( \psi = \varphi_2 - \varphi_1 \) using the difference \( t_1 - t_2 \) from the following equation:

\[
\frac{\omega t_2 + \varphi_2}{\varphi} = \frac{\omega t_1 + \varphi_1}{\varphi}
\]

\[
\psi = \varphi_2 - \varphi_1 = \omega (t_1 - t_2) = 2\pi \frac{t_1 - t_2}{T}.
\]

In the Table 2 is few examples of the relationship between the phase shift \( \psi \) and corresponding values of the time shift \( t_1 - t_2 \) expressed by the means of fractions of the period \( T \).

<table>
<thead>
<tr>
<th>( \psi ) [rad]</th>
<th>( t_1 - t_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \pm \pi/6 )</td>
<td>( \pm T/12 )</td>
</tr>
<tr>
<td>( \pm \pi/4 )</td>
<td>( \pm T/8 )</td>
</tr>
<tr>
<td>( \pm \pi/3 )</td>
<td>( \pm T/6 )</td>
</tr>
<tr>
<td>( \pm \pi/2 )</td>
<td>( \pm T/4 )</td>
</tr>
</tbody>
</table>

Table 2